We investigate the vector plasmonic lattice solitons (PLSs) in nonlinear graphene-pair arrays (GPAs) consisting of periodically arranged double graphene sheets, which are spatially separated. There are two dispersion bands for the Bloch modes in the array due to the coupling of surface plasmon polaritons (SPPs) between the graphene pairs. The vector PLSs composed of two components originate from the nonlinear interaction of Bloch modes in different bands. Both components undergo mutual self-trapping through the balance between diffraction and self-focusing nonlinearity of graphene. Thanks to the strong confinement of SPPs, the vector PLSs can be squeezed into a lateral width of $\sim \lambda/100$. The study provides a promising approach to all-optical control on a deep-subwavelength scale.

Optical spatial solitons, the stationary propagation of a light beam with a constant transverse profile, are formed as the diffraction and nonlinear effects reach a balance in a nonlinear optical system [1,2]. As a special type of spatial soliton, vector solitons have attracted much attention for potential applications in developing light-control-light techniques [3–5]. The vector solitons have at least two mutually influenced components with both being conventional spatial solitons. Also, each component originates from an eigenmode of the considered optical system and is modulated by the nonlinear Kerr effect due to the strong electric field intensity of all components including itself. In order to ensure robust stationary propagation, the components of vector solitons are usually incoherent and are of different frequencies or different polarizations [3]. Recent progress has been made in spatial solitons by using dielectric and metallic waveguide arrays [6–11], where the light diffraction can be flexibly engineered [12]. Furthermore, the solitons in metallic waveguide arrays, namely, plasmonic lattice solitons (PLSs), possess significantly small lateral size, as they are formed by surface plasmon polaritons (SPPs), which are strongly concentrated on metal surfaces [13,14]. The vector PLSs also have been demonstrated in metallic waveguide arrays and may find potential applications in subwavelength all-optical technology [9].

We have demonstrated that periodic graphene sheet arrays are able to play a similar role of metallic waveguide arrays in tailoring SPP diffraction [15,16]. However, the SPP mode in graphene is substantially different from that on metallic film. Apart from the common asymmetric SPP mode, there should be a long-range SPP mode on the metallic film, which corresponds to a plane wave in graphene. The difference is also applied for array structures. There are usually two dispersion bands corresponding to the propagating SPP Bloch modes in metallic waveguide arrays but only one nontrivial band in graphene sheet arrays. The other one refers to trivial plane waves due to the 2D feature of graphene. On the other side, SPPs on graphene exhibit stronger confinement, lower propagation loss, and flexible tunability in comparison with metals [17,18]. These features make graphene a competitive alternative to metals in manipulating SPPs.

In this work, we investigate the vector PLSs in nonlinear graphene-pair arrays (GPAs) considering that graphene is also an admirable nonlinear material [19–21]. Importantly, the structure is composed of periodic graphene pairs instead of single-layer graphene. A graphene pair consisting of two separated parallel graphene sheets can support symmetric and antisymmetric propagating SPP modes [22–24]. Consequently, there should be two dispersion bands for the GPAs. The vector PLSs are composed of two components belonging to different bands with different frequencies. The stationary propagation of vector PLSs is obtained by achieving the balance between the self-focusing nonlinearity of graphene and SPP diffraction. We also study the vector PLSs shaping by varying the chemical potential of graphene.

We start from investigating the linear properties of GPAs. As shown in Fig. 1, the structure is composed of alternative graphene sheets and the dielectric layers, which are arranged periodically along the x axis. The period of structure is $d = d_1 + d_2$ with $d_1$ representing the spacing of graphene in each period and $d_2$ the side-by-side distance of adjacent graphene pairs. The dielectric relative permittivity is denoted by $\varepsilon_r$. The surface conductivity $\sigma_g (\lambda, \mu, \tau)$ of graphene is governed...
In contrast with the metal-dielectric arrays [9], the bandgap is where the Bloch momentum is given by

\[ k = \frac{2\pi n}{\lambda} \]

in air, with the positions of graphene sheets denoted by the red dashed lines.

The dispersion relation, the dependence of the propagation constant \( \kappa \) on the lateral Bloch momentum, is shown in Fig. 2(a). The transverse magnetic field and electric field, \( H_y \) and \( E_x \), compose the eigenvector, while the propagation constant \( \kappa \) is the eigenvalue. In the calculation, graphene is treated as a thin film with an equivalent thickness \( \Delta \approx 1 \) nm. Then, the relative equivalent permittivity of graphene could be given by \( \epsilon_g = 1 + i\sigma_g/(\epsilon_0 \omega \Delta) \) [29]. In the linear case, the calculated wave vectors of SPPs are given by \( k_x = 50.05 + 0.65i \) \( \mu \) at \( \lambda_1 = 9.8 \mu m \) and \( k_x = 40.52 + 0.68i \) \( \mu \) at \( \lambda_2 = 10.2 \mu m \). The values coincide fairly with the exact results by solving Eq. (1). The method developed here aims at dealing with nonlinear systems.

Taking into account the nonlinearity, the surface conductivity of graphene can be written as \( \sigma_g = \sigma_{g, linear} + \sigma_{NL}(|E_z1|^2 + |E_z2|^2) \). The nonlinear conductivity \( \sigma_{NL} \) is given by [19,20]

\[ \sigma_{NL} = -\frac{3}{8} \frac{e^2}{\hbar^2} \left( \frac{V_F}{\mu} \right)^2 \left( \frac{\mu_c}{\omega} \right) \]

where \( \omega \) denotes the frequency and the Fermi velocity \( V_F \approx c/300 \). It is obvious that the equivalent permittivity of graphene is intensity-dependent. Considering the nonlinearity of graphene, Eq. (2) turns into a nonlinear eigenproblem, which could be solved by using the self-consistent method [30]. Two localized fields each having a Gaussian distribution are adopted as initial values for the iterative calculation. We set \( 80 V^2/\mu m^3 \) as the maximum intensity of the first component and \( 100 V^2/\mu m^2 \) for the second. In each step of the iteration, the permittivity of graphene is updated with the solved field distribution of the previous step. Repeating the process of solving the field distribution of the two components and updating the permittivity, we can obtain the transverse distribution of the vector PLs until the eigenvalues are stable. The results are shown in Fig. 3. From Fig. 3(a), the first component (\( \lambda_1 = 9.8 \mu m \)) has a similar but localized profile in

\[ \cos(\varphi) = \cosh(\kappa d) + \zeta \sinh(\kappa d) + \frac{e^2}{2} \sinh(\kappa d_1) \sinh(\kappa d_2), \]

where the Bloch momentum is given by \( \varphi = k_x d \) with \( k_x \) the Bloch wave vector along the \( x \) direction, \( \kappa = (k_0^2 - \epsilon_d k_0^2)^{1/2} \), \( k_0 = 2\pi n/\lambda \) is the propagation constant, and \( \zeta = i\kappa \sigma_g/(\epsilon_0 k_0) \) with \( \eta_0 \) the impedance in vacuum. The parameters are given by \( d_1 = 60 \) nm, \( d_2 = 80 \) nm, \( \epsilon_d = 1 \), \( \mu_c = 0.15 \) eV, and \( \tau = 0.5 \) ps [26,27].

The dispersion relation, the dependence of the propagation constant \( k_z \) on the lateral Bloch momentum, is shown in Fig. 2(b) for the wavelength \( \lambda_1 = 9.8 \mu m \) and \( \lambda_2 = 10.2 \mu m \). In contrast with the metal-dielectric arrays [9], the bandgap is determined by the edge of the Brillouin zone for GPAs.
comparison with the Bloch mode shown in Fig. 2(c). It could be viewed as nonlinear counterpart of the Bloch mode in the Brillouin center of the first band. The intensity distribution of the first component, \( |E| = |E_1|^2 + |E_2|^2 \), is illustrated in Fig. 3(b). The FWHM of the first component is about 0.22 \( \mu \)m or 0.02\( \lambda \). The sharp peaks of the intensity reveal the strong confinement of SPPs on the graphene sheets. The transverse electric field and the intensity of the second component are shown in Figs. 3(c) and 3(d). The effective width of the vector PLSs for the two components is given by [9,10]

\[
\psi_{1,2} = \left( \int x^2 |E_{1,2}|^2 dx / \int |E_{1,2}|^2 dx \right)^{1/2}.
\]

The calculated results are \( \psi_1 = 0.08 \mu \)m and \( \psi_2 = 0.13 \mu \)m, corresponding to 0.008\( \lambda_1 \) and 0.013\( \lambda_2 \), respectively. Because the band curve of the second component has a larger curvature than that of the first component, it has a little bit larger width.

To shed more light on vector PLS, we also investigate their propagation by using the modified split-step Fourier beam propagation method [8]. The propagation of vector PLS without considering the loss of graphene is shown in Figs. 4(a) and 4(b). It shows that the two components of the vector soliton remain constant lateral profiles all the time during propagation. If one of the two components is absent, the other has to suffer remarkable diffraction, as shown in Figs. 4(c) and 4(d). A stationary propagation of vector PLS requires both components at work. In other words, the two components influence each other simultaneously. The propagation of one component can be controlled by the other one. Taking the loss into consideration, the propagation of the vector PLS is illustrated in Figs. 4(e) and 4(f). The propagation distances of both components are roughly the same. The propagation distance of the second component is a little bit smaller than that of the first one because the loss rests with the original linear eigenmodes, as shown in Fig. 2(b). The propagation loss of the first component is smaller than that of the second one. Thereafter, the propagation length of the vector PLSs is mainly determined by the second component.

The propagation constants of the two components versus the peak intensity of the second component denoted by \( I_2 \) are plotted in Figs. 5(a) and 5(b). One sees that the propagation constants of both components increase as \( I_2 \) increases. For a fixed value of \( I_2 \), the propagation constants also increase as the peak intensity of the first component \( I_1 \) increases. Considering that the second component, originating from the second band, is located in the bandgap, the intensity \( I_2 \) should reach the maximum at the bottom of the first band for each value of \( I_1 \). The limitation of \( I_2 \) also appears for the first component as \( I_1 \) varies, although the first component originates from the first band and lies in the semi-infinite gap above, as shown in Fig. 5(a). When the intensity \( I_1 = 0 \) or \( I_2 = 0 \), the vector PLSs turn to scalar solitons. It should be mentioned that there is a bound state for the first (second) component even if its intensity approaches to zero, which results from the nonlinear mutual self-trapping effect induced by the second (first) component.

The influence of \( I_2 \) on the effective width is shown in Figs. 5(c) and 5(d). The effective width of the first component
decreases as the intensities \( I_1 \) and \( I_2 \) increase. When \( I_1 > 60 \, \text{V}^2/\mu\text{m}^2 \) or \( I_2 > 120 \, \text{V}^2/\mu\text{m}^2 \), the value of \( w_1 \) almost keeps at constant, as shown in Fig. 5(c). In this circumstance, the first component has concentrated into the region of a single graphene pair with a width about 0.06 \( \mu\text{m} \) and can no longer be squeezed. As for the second component shown in Fig. 5(d), the effective width decreases first, and then increases after reaching a minimum of \( w_2 = 0.12 \, \mu\text{m} \). The minimum is almost independent on \( I_1 \). The width of the second component reaches its minimum as its propagation constant locates in the middle of the bandgap. As \( I_2 \) increases further, the propagation constant approaches to the bottom of the first band and the width increases.

We also investigate the influence of chemical potential. The variation of band structure with chemical potential for the linear case is shown in Figs. 6(a) and 6(b). Here we only provide the first (second) band for the first (second) component as \( \lambda_1 = 9.8 \, \mu\text{m} \) (\( \lambda_2 = 10.2 \, \mu\text{m} \)). Generally, the propagation constants corresponding to different components decrease as the chemical potential increases. From Fig. 6(a), the curvature at the Brillouin center increases as the chemical potential increases, where the first component is growing. As a result, the first component becomes wider as the chemical potential increases, as shown in Fig. 6(c). The situation is roughly the same for the second component but for a larger beam width, as shown in Figs. 6(b) and 6(d). The reason lies in the fact that the curvature of the second band is remarkably larger than the first one, indicating stronger diffraction of SPPs. It should be noted that the maximum intensities of two components are fixed at \( I_1 = 80 \, \text{V}^2/\mu\text{m}^2 \) and \( I_2 = 100 \, \text{V}^2/\mu\text{m}^2 \), respectively. By comparing the variation of two components, the second one is more sensitive to the chemical potential than the first one in the range of \( \mu = 0.15–0.22 \, \text{eV} \). As the chemical potential increases further, strong coupling occurs, and the case is out of our consideration scope, as shown in Fig. 6(b).

In conclusion, we have investigated the vector PLSs in nonlinear graphene-pair arrays. Every graphene pair of GPAs consisting of two separated parallel graphene sheets can support symmetric and antisymmetric SPP modes. Consequently, there should be two bands of SPP Bloch modes in GPAs. The components of the vector solitons have different frequencies and arise from Bloch modes of two bands in GPAs. They undergo mutual self-trapping through the balance between SPPs diffraction and the nonlinearity effect of graphene. Due to the strong confinement of SPPs in graphene, the vector PLSs manifest a deep-subwavelength width. By varying the chemical potential of graphene, the effective width of vector PLSs can be flexibly tuned. This study may find applications in optical switches, optical circuits, and soliton-based navigation on a deep-subwavelength scale.

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