Single attosecond pulse generation by nonlinear Thomson scattering in a tightly focused intense laser beam

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The relativistic nonlinear Thomson scattering of a tightly focused intense laser pulse by an electron is investigated, and the temporal and spectral characters of the radiation are discussed. In a tightly focused laser pulse with an intensity of approximately $10^{20}$ W/cm$^2$ and a pulse duration of 20 fs, the electron is scattered away from the focus quickly by the ponderomotive force and therefore the radiation emitted at the focus is much higher than that at other regions. As a result, a single ultrashort pulse of 3.8 as is generated and its corresponding spectrum is broadened to 200 orders of the frequency of the driving laser. With increasing the laser intensity, the signal-to-noise of the radiated pulse increases, and the pulse duration decreases. Moreover, the phase behavior of the spectral components and the dependence of the radiated power on the laser intensity are discussed. © 2006 American Institute of Physics. [DOI: 10.1063/1.2164459]

I. INTRODUCTION

Ultrashort electromagnetic pulses, especially the single attosecond (as) pulse, are of great interest for the potential application of investigating and controlling the ultrafast processes.1 Until now various schemes have been explored for the generation of attosecond pulse, such as the schemes utilizing high order harmonics,2-5 stimulated Raman scattering,6 and molecular modulation.7,8 However, these methods have in theory a tendency to produce a train of closely spaced pulses rather than a single pulse. In experiment, Paul et al.9 observed a train of 250 as pulses with the photon energy of a few 10 eV. Further, the generation of a single attosecond pulse has gained much interest. Kien et al.10 investigated the high order harmonics generated from He atoms illuminated by an ultrashort laser pulse, and by using a spectral filter to select only several frequency components in the spectrum, a single attosecond pulse is generated in the laser with a pulse duration of approximately 5 fs. This has been experimentally demonstrated by Hentschel et al.11 they observed a 650 as pulse at 90 eV photon energy in a laser with a pulse duration of 7 fs. Corkum et al.12 suggested that by using laser fields with a time-modulated degree of ellipticity, it should be possible to generate a single pulse from the attosecond pulses train.

The radiation characters of the nonlinear Thomson scattering have been investigated in theories13-18 and experiments.19 For the possible application as x-ray sources,20,21 most of the above-mentioned work focuses their attention on the spectral characters of the radiation, whereas the temporal characters have not attracted enough attention. Recently, Lee et al.22 investigated the temporal characters of the radiation in the relativistic nonlinear Thomson scattering of a planar electromagnetic wave. A train of attosecond pulses with photon energy from 100 to 600 eV are obtained.

Further, by taking into account the effect of multielectrons, they proposed a method to enhance the radiation power.23 However, the laser beams used in most experiments are focused beams and can be focused down to a few microns now.24 Although, in the tightly focused laser case, the planar wave approximation is invalid. In the current paper, we take into account the evolution of the beam waist, and investigate the radiation characters of the nonlinear Thomson scattering in the nontightly and tightly focused Gaussian laser pulse. The spatial distribution, temporal and spectral characters of the radiation are discussed, and the results show that Thomson scattering in the tightly focused laser beam has a potential to produce a single attosecond pulse. Further, the phase behavior of the spectral components and the dependence of the radiation power on the laser intensity are discussed.

II. FORMULATION

We consider a circularly polarized Hermite-Gaussian (0,0) mode laser pulse with a beam waist of $b_0$ and a Gaussian pulse shape of $\exp(-\eta^2/L^2)$ where $\eta=z-t$. In this definition, the space and time coordinates are normalized by $k_0^{-1}$ and $\omega_0^{-1}$ where $\omega_0$ and $k_0$ are the laser frequency and wave number, respectively. To take into account of the evolution of the focused laser pulse, we adopt the similar method used in Ref. 26 to describe the laser field which is satisfactory for describing the electron dynamic and the radiation in our case. The transverse components of the vector potential are expressed as:

$$a_\perp = a_L \hat{a},$$

$$a_L = a_0 \exp(-\eta^2/L^2 - \rho^2/b_0^2)(b_0/b),$$

where $\hat{a}=(\cos(\phi)\hat{x} + \sin(\phi)\hat{y})/\sqrt{2}$, $\rho^2=x^2 + y^2$, $b=b_0(1 + z^2/t_0^2)^{1/2}$ is the radius of the spot size at position $z$, $a_0$ is the laser peak amplitude normalized by $mc^2/e$, $m$ and $e$ are the electron mass and charge, $t_0=b_0^2/2$ is the corresponding Rayleigh length, $\phi = \phi_p - \phi_G - \phi_0 + \phi_R$, where $\phi_p = z - t$, $\phi_R$
\begin{align}
\phi_R &= \tan^{-1}(z/z_f), \quad \phi_R = \frac{x^2 + y^2}{2R(z)}, \quad R(z) = z(1 + z_f^2/z^2). \quad \text{In the calculation, the time } t \text{ evolves from } -\infty \text{ to } \infty \text{ to include the whole laser pulse. The longitudinal component of the vector potential is expressed as}^2,5,6
a_z &= a_L \left(- \frac{2x}{b_0b} \sin(\phi + \theta) + \frac{2y}{b_0b} \cos(\phi + \theta)\right),
\end{align}

where \( \theta = \pi - \tan^{-1} z/z_f. \)

The motion of the electron in an intense laser field is described by the following equations:

\begin{align}
d_t(\mathbf{p} - \mathbf{a}) &= - \nabla_\mathbf{u}(\mathbf{u} \cdot \mathbf{a}), \\
d_t \gamma &= \mathbf{u} \cdot \mathbf{\dot{a}},
\end{align}

where \( \mathbf{a} \) is the vector potential expressed by Eqs. (1)–(3), \( \mathbf{u} \) is the electron velocity normalized by \( c, \quad \mathbf{p} = \gamma \mathbf{u} \) is the electron momentum normalized by \( mc, \quad \gamma = (1 - \mathbf{u}^2)^{-1/2} \) is the relativistic factor, \( \nabla_\mathbf{u} \) in Eq. (4) acts on \( \mathbf{a} \) only.

The angular distribution of the radiated power detected far away from the electron in the direction \( \mathbf{n} \) (see Fig. 1) can be calculated as

\begin{equation}
\frac{dP(t)}{d\Omega} = |A(t)|^2,
\end{equation}

\begin{equation}
A(t) = \left[ \frac{\mathbf{n} \times ((\mathbf{n} - \mathbf{u}) \times d\mathbf{u})}{(1 - \mathbf{u} \cdot \mathbf{n})^3} \right]_{\text{Ret}},
\end{equation}

where \( dP(t)/d\Omega \) is normalized by \( e^2 \omega_0^2 / 4\pi c \), the subscript Ret means that the quantities on the right-hand should be evaluated at the retarded time \( t' \), \( t' \) is related to \( t \) by \( t = t' + x_0 - \mathbf{n} \cdot \mathbf{r} \) where \( x_0 \) is the distance from the detector to the origin, \( \mathbf{r} \) is electron’s displacement. Solving Eqs. (4) and (5) with numerical methods, the electron motion in the laser field can be obtained. Then the radiation emitted from the interactions of a focused laser pulse with an electron can be calculated from Eqs. (6) and (7). Further, the angular spectral intensity (spectral intensity per solid angle) can be calculated by the Fourier transform of \( A(t) \)

\begin{equation}
\frac{d^2I}{dd\omega d\Omega} = 2|A(\omega)|^2,
\end{equation}

\begin{equation}
A(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(t) e^{-i\omega t} dt.
\end{equation}

III. RESULTS AND DISCUSSIONS

In this section, the radiation characters of the nonlinear Thomson scattering are discussed. Figure 1 shows the geometry for the scattering process. In this figure, the Gaussian laser pulse propagates in the +\( \mathbf{z} \) direction. The electron is accelerated at the focus, then the energetic electron emits radiation in the direction \( \mathbf{n} \). The angular distributions of the maximum radiated power for various intensities in a nontightly focused (the beam waist \( b_0 = 30 \alpha \), where \( \lambda = 800 \) nm is the laser wavelength) and a tightly focused (the beam waist \( b_0 = 3 \alpha \)) laser beam are shown in Fig. 2(a) and Fig. 2(b), respectively. The pulse width \( L = 5 \lambda \) (the corresponding pulse duration is approximately 20 fs). For comparison, each radiated power is normalized by its own maximum. It is clearly seen from Fig. 2 that the radiation is emitted toward +\( \mathbf{z} \) direction with a small angle. As the laser intensity increases, the polar angle \( \theta_m \), at which the radiation reaches its maximum, and the angular divergence \( \Delta \theta \) decrease. Comparing Fig. 2(b) with Fig. 2(a), we know that with respect to the same laser intensity, \( \theta_m \) depends slightly on the laser beam waist, but the angular divergence \( \Delta \theta \) is larger in the tightly focused laser case.

The electron trajectory in a circularly polarized laser pulse is a helix which is shown in Fig. 3(a). The laser intensity is \( I = 10^{20} \) W/cm\( ^2 \) and the other parameters are the same as those in Fig. 2(a). As shown in Fig. 3(a), the transverse quivering amplitude is approximately \( \lambda \), much less than the beam waist \( b_0 \), whereas the longitudinal drift distance is only approximately 0.3\( z_f \), also much less than the Rayleigh length. Therefore, the nontightly focused laser acting on the electron is near to a planar wave. On the other hand, the electron is oscillating in the laser beam and then the energetic electron will emit radiation. Since the laser intensity changes slightly at the focus, the emitted radiation power changes slightly as well, and then a train of pulses are generated. The time history of the radiated power per solid angle

\begin{figure}
\centering
\includegraphics[width=0.8\textwidth]{figure1}
\caption{Schematic geometry for the propagation of an incident driving laser and the radiation. The azimuthal angle \( \phi = 0 \) in our case.}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=0.8\textwidth]{figure2}
\caption{Angular distribution of the maximum radiation power for various intensities in (a) a nontightly focused laser beam (the beam waist \( b_0 = 30 \alpha \)) and (b) a tightly focused laser beam (the beam waist \( b_0 = 3 \alpha \)). The pulse width \( L = 5 \lambda \). For comparison, the radiations are normalized by their own maximum.}
\end{figure}
in the direction of the maximum radiation is shown in Fig. 3(b). The emitted pulse with the highest peak power is magnified in Fig. 3(c) to manifest the pulse shape. It is clearly seen from Fig. 3(b) that a train of attosecond pulses with almost constant interval of approximately 5 fs are generated and the pulse duration is approximately 2.5 as. The peak powers of the emitted pulses change slightly. In order to describe this change quantitatively, we define the “signal-to-noise” as the ratio of the highest peak power versus the second-highest one. As shown in Fig. 3(b), the signal-to-noise is approximately 2 which is a little higher compared with the radiation in the case of a planar wave. The above results indicate that the nontightly focused laser acting on the electron is near to a planar wave and therefore the radiation in the nontightly focused laser beam shows the similar characters with that in the planar wave.

It is shown by Fig. 3 that the planar wave approximation is valid when the beam waist \( b_0 \gg \lambda \). But in the current discussion, the laser beam can be focused down to a few microns, say 3\( \lambda \) or even to \( \lambda \). In this case, the interaction scenario is much different from the previous situation. The trajectory of the electron in a tightly focused laser pulse is shown in Fig. 4(a). The beam waist \( b_0 = 3\lambda \) and other parameters are the same as those in Fig. 3. It is shown by Fig. 4(a) that the transverse quivering amplitude is approximately 2\( \lambda \), comparable to \( b_0 \); whereas the longitude drifting distance is approximately 20\( \tau_f \), much larger than the Rayleigh length.

So in this process, the radiation is stronger at the focus since the laser intensity is higher here; whereas the radiation is weaker at other regions where the laser field becomes much weaker. As a result, the signal-to-noise of the radiation increases, and then a single attosecond pulse can be obtained. The time history of the radiated power per solid angle in a tightly focused laser pulse in the maximum radiation direction is plotted in Fig. 4(b), and the pulse with the highest peak power is magnified in Fig. 4(c). As shown in Fig. 4(b), a train of attosecond pulses are obtained. But the highest peak power is approximately 10 times of the second-highest one, even much higher than the others. Practically, it can be regarded as a single attosecond pulse. Comparing Fig. 4(c) with Fig. 3(c), we know that the peak power of the radiation is lower in the case of the tightly focused laser. It is because that in the tightly focused laser case, the laser intensity becomes weaker far away from the focus. The total energy is smaller than that of a nontightly focused laser, thus the radiation is weaker. The pulse duration of the highest peak is 7 as, which is longer than that in the nontightly focused laser pulse. It is due to the larger angular divergence \( \Delta \theta \) in the tightly focused laser case.

For laser beams with a higher intensity, the electron will be scattered away from the focus more quickly because of the stronger ponderomotive force. The radiation at the focus will be much stronger than that at other regions, then a single attosecond pulse with higher signal-to-noise can be obtained.

FIG. 3. (a) The electron trajectory in a nontightly focused circularly polarized laser pulse. (b) The time history of the radiated power per solid angle in the direction of the maximum radiation [see Fig. 2(a)]. (c) To investigate the pulse shape, the highest peak in (b) is magnified. The parameters are \( f = 1 \times 10^{20} \text{ W/cm}^2 \), \( b_0 = 30\lambda \), and \( L = 5\lambda \).

FIG. 4. (a) The electron trajectory in a tightly focused circularly polarized laser pulse. (b) The time history of the radiated power per solid angle in the direction of the maximum radiation. (c) To investigate the pulse shape, the highest peak in (b) is magnified. The beam waist \( b_0 = 3\lambda \) and other parameters are the same as those in Fig. 3.
The time history of the radiated power per solid angle in the maximum radiation direction in a higher-intensity laser field is shown in Fig. 5. The laser intensity $I = 4 \times 10^{20} \text{ W/cm}^2$ and the other parameters are the same as those in Fig. 4. It can be seen from Fig. 5 that the highest peak is well-marked, the second-highest one marked by the arrow is much lower, and the other peaks are too low to be distinguished. In this case, the signal-to-noise increases to 107. The second-highest peak power is less than 1% of the highest one and the other ones are even lower, then a single pulse with the pulse duration of 3.8 as is obtained. The spectrum of the above pulse is shown in Fig. 6(a). As shown in Fig. 6(a), the spectral intensity increases steeply at the beginning and reaches its maximum when $\omega/\omega_0=8$, then the intensity decreases gradually. It decreases to one-tenth of its maximum at $\omega/\omega_0=200$; beyond that the spectral intensity decreases slowly and keeps approximately 1% of its maximum. Compared to the spectrum of the radiation in a planar wave, the cutoff of the spectrum in this case is much lower. It is because that the motion of the electron in a tightly focused laser is less relativistic. In Fig. 6(b), the phases of the spectral components of this attosecond pulse are plotted. As shown in Fig. 6(b), even though the harmonics are not phase locked,\(^4\) it is not completely random. The phases behavior of the spectral components from $100\omega_0$ to $200\omega_0$ is more regular than the other ones.

Comparing Fig. 4 with Fig. 5, we may conclude that a single attosecond pulse with a higher signal-to-noise and shorter pulse duration can be obtained in the laser field with a higher intensity. This can be seen from Fig. 7 more clearly. In Fig. 7, the laser intensity dependence of the signal-to-noise and pulse duration of the emitted pulse is plotted. As the laser intensity increases, the signal-to-noise increases slowly when $I < 10^{20} \text{ W/cm}^2$, beyond that it increases steeply. At $I = 1.6 \times 10^{21} \text{ W/cm}^2$, a single attosecond pulse with a signal-to-noise of 523 and a pulse duration of 2.3 as is generated. This can be explained by the fact that the ponderomotive scattering\(^{28}\) occurs when $I > 10^{20} \text{ W/cm}^2$. In this case, the quivering amplitude of the electron exceeds the beam waist, so that the electron is scattered quickly away from the focus. Then the laser intensity decreases, the radiation becomes much weaker than that at the focus. Therefore the signal-to-noise increases significantly. Further, we can see from Fig. 7 that the pulse duration decreases with the increasing of the incident laser intensity.

Figure 8 shows the laser intensity dependence of the radiation power in the direction of the maximum radiation. The radiation power increases nonlinearly with the increasing of the laser intensity. Compared to the result in a planar wave,\(^{22}\) the distinct characteristic shown in Fig. 8 is that the laser intensity dependence of the radiation power shows two scenarios. Although $I < 10^{20} \text{ W/cm}^2$, the scaling to the laser intensity can be roughly estimated as $dP/d\Omega \propto I^{3.7}$; whereas $I > 10^{20} \text{ W/cm}^2$, $dP/d\Omega \propto I^{1.5}$. This can be explained as follows: the electrons are scattered away before the laser intensity reaches its maximum when $I > 10^{20} \text{ W/cm}^2$. Thus, the radiation power increases less quickly with the increasing of the laser intensity.

Moreover, numerical results show that the radiation
The problems in the multielectrons case still have been investigated in the recent works, arguments that Thomson scattering in a tightly focused laser beam has a potential to produce a single attosecond pulse instead of a train of attosecond pulses. With increasing the laser intensity, the signal-to-noise and peak power of the radiated attosecond pulse increase, and the pulse duration decreases. For a tightly focused driving laser with an intensity of $4 \times 10^{20}$ W/cm$^2$ and a pulse duration of 20 fs, a single attosecond pulse with a pulse duration of 3.8 as and a signal-to-noise of 107 is obtained. The corresponding spectrum of this pulse is broadened to 200$\omega_0$. Moreover, even though the harmonics are not phase locked, the phases behavior of the spectral components from 100$\omega_0$ to 200$\omega_0$ is more regular than the other ones.

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