Spectral distributions of harmonic generation from electron oscillation driven by intense femtosecond laser pulses

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Abstract

The characteristics of harmonic radiation due to electron oscillation driven by an intense femtosecond laser pulse are analyzed considering a single electron model. An interesting modulated structure of the spectrum is observed and analyzed for different polarization. Higher order harmonic radiations are possible for a sufficiently intense driving laser pulse. We have shown that for a realistic pulsed photon beam, the spectrum of the radiation is red shifted as well as broadened because of changes in the longitudinal velocity of the electrons during the laser pulse. These effects are more pronounced at higher laser intensities giving rise to higher order harmonics that eventually leads to a continuous spectrum. Numerical simulations have further shown that by increasing the laser pulse width broadening of the high harmonic radiations can be limited.

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1. Introduction

Recently advances in the short-pulse and high intensity laser technology has enabled researcher to explore new and sometimes unexpected physics, which can occur in laser matter interactions as intensities approach $10^{18}$ W/cm². An important tool in study of such interactions is the phenomena of harmonic emission. Harmonic generation (HG) has received wider attention for many years in the studies of oscillations of electrons in the intense laser fields [1–11], and in discussions of laser plasma interactions [12,13]. Krafft has investigated the general formulas of the Thomson scattering by a single electron in an analytical way and found a new phenomenon that is called “ponderomotive broadening” [6]. Scattering of continuous wave of laser light at higher intensity is well known and one can obtain solution of the classical motion of an electron in a plane wave [14] for the case of constant continuous illumination, and apply it for calculating the harmonic power.

In this paper, we present a solution of the general problem of classical motion and radiation of a single electron interacting with an intense femtosecond Gaussian laser pulse which covers more cases than that of Krafft [6]. The modulated structure of the energy spectrum will be presented and analyzed for different incident polarization and in different observation directions. It is also discovered the same phenomenon that with a laser intensity of $10^{18}$ W/cm², the oscillation of electron, due to their relativistic motion, will generate harmonics emission. The spectrum of harmonic emission is broadened and red shifted. Such red shift and ponderomotive broadening is...
especially pronounced at higher harmonics. With increasing the width of the laser pulse, broadening of the electron’s harmonic radiation can be limited. The effects discussed in this paper may be used to explain the broadening and red shift of harmonic radiations from underdense plasma irradiated by femtosecond intense laser pulses.

2. The interaction model and formulation

For a Gaussian laser pulse, the vector potential can be expressed as

$$\mathbf{a}(\eta) = a_0 \exp(-\eta^2/2L^2) [\cos(\eta) \mathbf{x} + \delta \sin(\eta) \mathbf{y}],$$

(1)

where $a_0$ is the peak amplitude normalized by $me^2/e$, $\eta = z - t$, $L = d/2$ and $d$ is the laser pulse width, $\delta$ is the polarization parameter with $\delta = 0$ for linear polarization and $\delta = \pm 1$ for circular polarization, respectively. In the above definitions, space and time coordinates are normalized by $k_0^{-1}$ and $\omega_0^{-1}$, respectively, and $\omega_0$ and $k_0$ are the laser frequency and wave number, respectively. $m$ and $e$ are the electron’s mass and charge, respectively.

The configuration of laser-electron interaction is shown in Fig. 1. We assume the laser pulse propagates along the $+z$ axis and an electron is initially stationary at the origin of coordinate. The radiation direction is $\mathbf{n} = \sin \theta \cos \phi \mathbf{x} + \sin \theta \sin \phi \mathbf{y} + \cos \theta \mathbf{z}$ (Fig 1). In these expressions $\theta$ and $\phi$ are the usual spherical coordinates with the $z$ axis aligned along the electron longitudinal motion.

The motion of an electron in an electromagnetic wave is described by the Lorentz equation [15]

$$d_t(p - a) = -\nabla_a (a \cdot u),$$

(2)

together with an energy equation

$$d_t \gamma = u \cdot \partial_t a,$$

(3)

where $u$ is the velocity of electron normalized by $c$, $p = \gamma u$ is the normalized momentum, $\gamma = (1 - u^2)^{-1/2}$ is the relativistic factor or normalized energy, and the $\nabla_a$ in (1) acts on $a$ only. Note that Eqs. (2) and (3) are exact.

As the solution of 1D wave equation, the normalized vector potential $a = a(\eta)$. The quantities describing electron motion are assumed to be functions of $\eta$ as well. With $\partial_z = \partial_\eta$ and $\partial_t = -\partial_\eta$, one gets from (2) and (3)

$$\gamma u_\perp = a, \quad \gamma (u_z - 1) = e,$$

(4)

$$\gamma = -(1 + \delta^2 + \delta^2)/2e,$$

(5)

where we have assumed the transverse velocity $u_\perp = 0$ when $a = 0$, $e$ is a constant of the motion to be determined by the initial conditions on the electron motion. The motion of the electron can be fully determined; the velocity and displacement can be expressed as functions of $\eta$

$$u_\perp = a/\gamma, \quad u_z = 1 + e/\gamma,$$

(6)

$$r_\perp = \frac{1}{a} \int a \, d\eta, \quad r_z = \frac{1}{2a^2} \int (\delta^2 - 1 - a^2) \, d\eta,$$

(7)

where $r_\perp$ and $r_z$ are the transverse and longitudinal displacement of the electron, respectively.

Electron in relativistic motion emits radiation, the radiated power per solid angle is given by [14]

$$\frac{dP(t)}{d\Omega} = \left[ \frac{\mathbf{n} \times (\mathbf{n} - \mathbf{u}) \times d\mathbf{u}}{(1 - \mathbf{n} \cdot \mathbf{u})^6} \right]_t,$$

(8)

where the radiation power is normalized by $e^2 \omega_0^2/4\pi c$ and $t'$ is the electron’s time or retard time. The relation between $t'$ and $t$ is given by

$$t = t' + R, \quad R \sim R_0 - \mathbf{n} \cdot \mathbf{r},$$

(9)

Fig. 1. Schematic diagram showing the interaction of an incident laser and a stationary electron, we assume the laser field propagate along $+z$ axis.

Fig. 2. Backward radiation frequency spectrum of the electron in a circular polarized Gaussian laser pulse for (a) $a_0 = 0.01$ and $d = 10\lambda_0$, (b) $a_0 = 0.5$ and $d = 10\lambda_0$. 
where $R_0$ is the distance from the origin to the observer and $\mathbf{r}$ is the position vector of the electron. Here the observation point is assumed to be far away the region of space where the acceleration occurs.

The energy radiated per unit solid angle per unit frequency interval is given by [14]

$$\frac{d^2I}{d\omega d\Omega} = s^2 \int_{-\infty}^{\infty} \mathbf{n} \times (\mathbf{n} \times \mathbf{u}) e^{i\omega(t - \mathbf{n} \cdot \mathbf{r})} d\tau,$$

(10)

where $\frac{d^2I}{d\omega d\Omega}$ is normalized by $e^2/4\pi^2 c$. $s = \omega_{sb}/\omega_0$, $\omega_{sb}$ is the frequency of the backward scattering radiation. Because of the symmetry of the electron orbit, the energy distribution is independent of $\phi$.

3. Results and discussion

Radiation characteristics are investigated for various laser peak intensities, for different laser pulse width ($d$), and for two different polarizations. A driving laser has a Gaussian envelope with a pulse duration of $d = 10\lambda_0$ (33 fs, $\lambda_0$ is the wavelength normalized by $k_0^{-1}$ and can be replaced by $2\pi$) and $d = 30\lambda_0$ (100 fs), and the central wavelength at 1 $\mu$m. A numerical method, based on integration of Eq. (10), has been used to investigate the energy radiated per unit solid angle per unit frequency interval of the electron. The electron is initially stationary in the lab frame, so $\gamma = 1$ and $\epsilon = -1$, and the harmonic frequency scale is set by the incident laser frequency without a Doppler shift.

Fig. 2 shows the energy radiated per unit solid angle per unit frequency interval of the electron in a circularly polarized Gaussian laser pulse at $\phi = 0^\circ$, $\theta = 180^\circ$ for $a_0 = 0.01$ and $d = 10\lambda_0$ (Fig. 2(a)), $a_0 = 0.5$ and $d = 10\lambda_0$ (Fig. 2(b)). One can see that there is only the fundamental frequency harmonic [7] in the electron’s radiation frequency spectrum of the backward radiation without red shift and broadening for $a_0 = 0.01$ (Fig. 2(a)), but with broadening and red shift occurring for $a_0 = 0.5$ (Fig. 2(b)). The broaden phenomenon is called “ponderomotive broadening” by Krafft [6]. As the laser pulse travels through, the electron is first accelerated, and then slowed down by the ponderomotive force from the laser pulse. These velocity shifts lead to frequency shifts in the emitted radiation, thus increasing the width of the observed spectrum. Because the electron spends a larger fraction of time in the “high-field” region of the laser pulse, it will radiate most and with maximum red shift.

Much more interesting is the result when a Gaussian laser pulse of linear polarization is considered. Fig. 3 gives four radiation frequency spectrum distributions of the electron in a linearly polarized Gaussian laser pulse at $\phi = 0^\circ$, $\theta = 180^\circ$ for (a) $a_0 = 0.1$ and $d = 10\lambda_0$ (Fig. 3(a)), (b) $a_0 = 0.5$ and $d = 10\lambda_0$ (Fig. 3(b)), (c) $a_0 = 1.0$ and $d = 10\lambda_0$ (Fig. 3(b)), (d) $a_0 = 0.5$ and $d = 30\lambda_0$. 

![Fig. 3. Backward radiation frequency spectrum of the electron in a linear polarized Gaussian laser pulse for (a) $a_0 = 0.1$ and $d = 10\lambda_0$, (b) $a_0 = 0.5$ and $d = 10\lambda_0$, (c) $a_0 = 1.0$ and $d = 10\lambda_0$, (d) $a_0 = 0.5$ and $d = 30\lambda_0$.](image-url)
and contrasting Fig. 3(b) with Fig. 3(d), one can clearly see on the electron’s backward harmonic radiation. Comparing and the effects above discussed can be exactly used to explain the results obtained in the experiment.

The conditions of our model are consistent with the recent experiment [6,12] and the effects above discussed can be exactly used to explain the results obtained in the experiment.

Fig. 3 also shows the influence of the laser pulse width \( d \) on the electron’s backward harmonic radiation. Comparing and contrasting Fig. 3(b) with Fig. 3(d), one can clearly see that broadening can be effectively limited with increasing the laser pulse width. By examining the harmonic peaks in detail, it is also found that the frequency shift to longer wavelength with the laser pulse width increasing.

One can see from Figs. 2 and 3 that there are many dips within the spectral peaks. By examining the dips in detail, the dips are due to destructive interference of the induced emission from various longitudinal positions within the laser pulse [16].

The emission spectrum at \( \phi = 0^\circ, \theta = 145^\circ \) is examined, a broaden feature, red shift relative to the exact harmonics frequency, is also observed. In Fig. 4(a) and (b), harmonics spectra are shown for circularly and linearly polarized driving laser pulse, respectively. One can see that both odd and even harmonics [7] exist in the radiation of the electron at \( \phi = 0^\circ, \theta = 145^\circ \) with broadening and red shift occurring.

From Fig. 4 one can see that changing the incident laser polarization from linear to circular does not affect the spectral shape.

4. Conclusions

We have presented some results from a general theory for calculating the energy spectral distribution of the photons generated in different directions from electron oscillation driven by two different polarized laser pulses. At \( \theta = 180^\circ \), the radiation spectra shape is different for circularly and linearly polarized driving laser pulse. While changing the incident laser polarization from linear to circular does not affect the spectral shape at \( \theta = 145^\circ \). It shows that the spectrum is broadened due to ponderomotive effects, which change the electron longitudinal velocity. It is a new phenomenon which is called “ponderomotive broadening” [6]. It is also found that long laser pulse width can effectively limit “ponderomotive broadening”. The results obtained in this paper are not consistent with previous theories in that finite laser pulse length is allowed simultaneously with high effective laser intensity. The effects obtained in this paper can be used to explain the broaden and redshift of harmonics from underdense plasma irradiated by femtosecond intense laser pulses.

Further, investigations are still required to include the effect of multielectrons; that is, electrons at different positions emit radiations with time interval in electrons’ view. The phase matching of the radiations from different electrons needs to be addressed.

References