Plasmonic routing in aperiodic graphene sheet arrays

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We discover deep-subwavelength, low-loss, and diffraction-free surface plasmon polariton (SPP) beam routing effects in aperiodic graphene sheet array (a-GSAs). The a-GSAs are constructed by varying either the interlayer spaces between graphene or the individual graphene chemical potentials. The SPP beams can be accelerated or decelerated in the a-GSAs, resulting in beam routing in different paths. The wave fronts of the beams are always parallel to the propagation direction, allowing the generation of transverse radiation pressure. All of these behaviors of SPPs are demonstrated by fully vectorial simulation and Hamilton optics analysis. © 2014 Optical Society of America

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cess of the graphene [100]. Additionally, the group and phase velocities of the SPP beams are orthogonal to each other. The propagation loss of SPPs in the a-GSAs could be much smaller than that in single-layer graphene.

Fig. 1. (a) Schematic of the a-GSA structure. (b) Wave vector (normalized by \( k_0 \)) of the collective SPPs in the periodic GSA versus the Bloch momentum (\( \phi = k_x d \)) in the first Brillouin zone for different periods. The solid and dashed curves refer to the real and imaginary parts of the normalized wave vectors, respectively. The normalized wave vector of SPPs in a single-layer graphene is marked with green arrows on the boundaries of the figure.

Light propagation in discrete optical systems composed of periodic waveguide arrays has attracted much attention in the past few decades [1–4]. By engineering the diffraction curve, which depicts the relation between the longitudinal wave vector and the Bloch momentum of the collective mode in the waveguide arrays, the light diffrac-
tion can be controlled artificially [5]. With respect to dielectric waveguide arrays, the diffraction curve abides by a cosine function of the Bloch momentum [2]. As the Bloch momentum reaches \( \pi/2 \), the diffraction curve becomes linear and the light beam in the array may ex-
perience diffraction-free propagation. For metallic wave-
guide arrays, the diffraction curve is reversed because of negative coupling [6]. In 2009, Verslegers et al. theoreti-
cally demonstrated that an aperiodic metallic waveguide array can focus light into a width as small as one-
hundredth of the wavelength in air [4].

Recently, we found that the diffraction relation of gra-
phene sheet array (GSA) is significantly different from that of traditional waveguide arrays [7]. Graphene is a two-dimensional material and can support transversely polarized surface plasmon polaritons (SPPs) [8,9]. As the period of the array is less than the plasmonic thick-
ness of the graphene [10,11], which quantifies the SPP field confinement in a single-layer graphene, SPPs in indi-
vidual graphene sheets undergo strong coupling with each other. In this situation, the diffraction curve of the collective SPP mode in the GSA becomes linear around the Brillouin zone center. The properties of SPPs propagating in such a system are still under investigation.

In this Letter, we will study SPPs propagating in aperi-
odic graphene sheet arrays (a-GSAs) in the case of strong coupling. The propagating route of SPPs in the a-GSAs can be controlled by varying the interlayer spaces between graphene or individual graphene chemical potentials. Because of the linear diffraction relation, the route of SPPs depends weakly on the Bloch momentum. Thus the traditional focusing of light is hard to be observed. Instead, the a-GSA can find application in deep-subwavelength optical routing [12,13]. Additionally, the group and phase
Fig. 1(b), where $k_z$ is the longitudinal wave vector and $\phi = k_z d$ is the Bloch momentum with $k_z$ the Bloch wave vector. The light wavelength in air is given by $\lambda = 10 \mu m$ and the chemical potential of graphene $\mu_g = 0.15$ eV. We assume room temperature (300 K) and the momentum relaxation time of electrons in graphene $\tau = 0.5$ ps. The plasmonic thickness of graphene [11] is given by $\xi = \sigma_0 / (i c_0 k_\|)$, where $\sigma_0$ is the surface conductivity of graphene and could be modeled by the Kubo formula [14]. $\eta_0$ is the impedance of air, and $k_0 = 2 \pi / \lambda$. We get $\xi = 46$ nm by using the above parameters. As $d < \xi$, strong coupling between graphene sheets occurs. In this situation, the diffraction relation becomes linear and has the form $k_z = \alpha^{-1/2} |k_z|$ around the Brillouin zone center [7], where $\alpha = \xi / d - 1$. As the dispersion relation is generally given by $\omega = \omega(k_x, k_z)$, the diffraction relation can be expressed as $\omega(k_x, k_z) - \omega_0 = 0$ at a fixed frequency $\omega_0$, where $k_x$ is an implicit function of $k_x$ and $dk_x / dk_z = -(\partial \omega / \partial k_z) / (\partial \omega / \partial k_x)$. The group velocity reads $\vec{v}_g = (\partial \omega / \partial k_z) \hat{\xi} + (\partial \omega / \partial k_x) \hat{k}_x$ and thus $\vec{v}_g, \vec{k} = k_x (\partial \omega / \partial k_x) + k_z (\partial \omega / \partial k_z)$. Because $k_z$ is proportional to $k_x$, we have $dk_x / dk_z = k_x / k_z$. As a result, $\vec{v}_g, \vec{k} = 0$. Since the phase velocity $\vec{v}_p$ has the same direction of the wave vector $\vec{k} = (k_x, k_z)$, the phase and group velocities are orthogonal to each other. The inference holds firmly under the conditions of strong coupling and paraxial limit, that is, the linear diffraction relation near the Brillouin zone center ($\phi \approx 0$).

Now we consider an a-GSA with the local period generally following $a(x) = ax^b$, that is, $d(x) = \xi / (ax^b + 1)$, where $a(> 0)$ and $b$ are arbitrary real numbers, referring to the distribution density and aperiodicity degree of graphene in the arrays, respectively. When $b = 0$, the structure becomes periodic. Figures 2(a) and 2(b) depict the magnetic field ($H_y$) distribution in the a-GSAs for $(a, b) = (4, -1)$ and $(1, 2)$. The choice of these values is to guarantee the paraxial condition of light propagation. The results are calculated numerically by using the finite-difference frequency-domain (FDFD) method [7,10], where graphene is treated as a thin metallic film with a thickness of 1 nm [15]. The incident waves are injected by exciting SPPs individually in each graphene sheet with the amplitude in the $n$th graphene following $A_n = A_0 \exp(-\alpha(x_n - x_0)^2 / 2w_0^2) \exp(i \phi_n)$, where $x_n$ stands for the location of graphene, $\phi_n$ is the phase of individual SPP mode in the $n$th graphene, $w_0 = 0.2 \mu m$ denotes the width of wave envelope in intensity, and $x_0 = 1 \mu m$ is the center of the incident field. The schematics of the incident $H_y$ field profiles are shown in the inset of Figs. 2(a) and 2(b). The phase difference of individual SPP mode between adjacent graphene sheets $\phi = -0.1 \pi$ is introduced to yield the local Bloch momentum. Other values of the local Bloch momentum will not change the propagation routes of the SPP beam as long as they are located in the linear regime of the diffraction relation. However, the loss can be significantly reduced by employing a smaller absolute value of $\phi$, according to Fig. 1(b). As shown in Fig. 2, both beams tend to propagate on the right-hand side of the $z$ axis. For $b = -1$, as illustrated in Fig. 2(a), the beam shows an analogous feature of a propagation route with the Airy beam, which tends to be accelerated during propagation [16]. On the contrary, the beam deceleration is also observed in Fig. 2(b) as $b = 2$. The accelerated beam suffers slight diffraction as the beam width increases slowly, while the decelerated beam is being narrowed during propagation. We also plot the Poynting vectors (black arrows) of the SPP beams. Note that the wave fronts of both accelerated and decelerated beams are always parallel to the propagation direction. Thus the direction of power flux is perpendicular to that of the phase propagation, verifying the orthogonality of the phase and group velocities. The beam can generate transverse radiation pressure perpendicular to the propagation direction that can drive external small particles [17]. It should be mentioned that the width of the SPP beam during propagation can further decrease when using a smaller incident beam width such as $w_0 = 0.1 \mu m$. However, in order to generate a relatively determinate Bloch momentum, the incident beam width should not be smaller than the mode width of a single-layer graphene [7].

To shed light on the behaviors of SPPs in the a-GSA, we employ Hamilton optics [18] to analyze the trajectory of the SPP beams. Hamilton optics adapts the analogy between the Hamilton principle in mechanics and the Fermat principle in optics to predict the beam route in a bulk medium as long as the local dispersion relation is known. Under the paraxial condition (linear regime of the diffraction curve), the Hamiltonian of the a-GSA is given by $H = -k_z = -c^{-1/2} |k_z|$. According to the optical Hamilton equations [4], the route of SPPs can be figured out by
It shows evidently that the SPP route only depends on the sign of \( k_z \). Since the GSA is a negative coupling system, there should always be \( k_z x < 0 \) [7]. As the beam propagates on the right-hand side of the \( z \) axis \( (x > 0) \), the lateral wave vector is always negative \( (k_z < 0) \). In this situation, the propagation route of the SPPs is independent on \( k_z \). In consequence, the solution of the SPP route is given by

\[
\begin{align*}
\frac{dx}{dz} &= -\alpha^{-1/2} \text{sgn}(k_z). \\
&= \begin{cases} \\
\alpha^{1/2}/ \sqrt{b/2 + 1} (a^{2b+1} - x_0^{2b+1}) & b \neq -2 \\
\alpha^{1/2} \ln \left( \frac{x}{x_0} \right) & b = -2. 
\end{cases}
\end{align*}
\]

(2)

From the propagation route of the SPPs, the ratio between the (group) velocities in the \( x \) and \( z \) directions is given by \( \nu_x/\nu_z = dx/dz \propto x^{-b/2} \). Because of the translational symmetry, the velocity of SPPs along the \( z \) direction remains constant. Consequently, the velocity along the \( x \) direction is increasing during propagation for \( b < 0 \). That is the reason we observe beam acceleration along the \( x \) direction in Fig. 2(a). On the contrary, the beam is decelerated for \( b > 0 \), as shown in Fig. 2(b).

The analytical results derived from Eq. (2) are shown in Figs. 2(c) and 2(d). The predicted SPP routes coincide well with the numerical data. As \( b = -1 \), the propagation route of SPPs is given by \( x = (z + 2a x_0^2)/(4a) \). The beam width obeys \( w \approx w_0[1 + z/(2\sqrt{ax_0^2})] \) as \( x_0 \gg w_0 \). It verifies the beam diffraction as the propagation distance increases. If the incident point \( x_0 \) approaches to zero, the beam width increases and a stronger diffraction should be observed. As \( b = 2 \), the beam width 

\[
w \approx w_0/\sqrt{2z/(\sqrt{ax_0^2} + 1)} + 1.
\]

It reveals that the decelerated beam is narrowed during propagation. For \( a = 1 \), the narrowing factor is given by \( s = w_0/w = \sqrt{2z/\sqrt{x_0^2} + 1} \). As \( x_0 = 1 \) \( \mu \)m and \( w_0 = 0.2 \) \( \mu \)m, for example, we get \( s = 2.6 \) and the beam width becomes \( w = 77 \) \( \text{nm} \) at \( z = 3 \) \( \mu \)m. The value is about 1/130 fold incident wavelength, which remarkably breaks through the diffraction limit. The average intensity of the SPP beam can be defined as \( I = (P_1/w) = s(P_1/w_0) \) with \( P_1 \) the total power of the beam at a certain distance. The power has to attenuate because of propagation loss, while the narrowing factor \( s \) increases with the propagation distance. Thus the average intensity of the decelerated beam can be sustained in a longer distance if the power loss remains at the same level with the accelerated beam.

Because of the translational symmetry along the \( z \) direction, \( k_z \) is conserved once the incident condition is specified [4]. Note that the incident interface is normal to the graphene sheets. The lateral component of wave vector \( (k_z) \) is real while \( k_z \) is complex [19]. Thus the total power loss is determined by the propagation distance in the \( z \) direction. As a result, the total power at a given distance can be written as \( P_t = \int P_0(x) \exp[-2 \text{Im}[k_z (x)]z] \text{d}x \), where \( P_0(x) \) is the incident power density at \( z = 0 \). Since the incident beam width is very narrow, we approximately have \( P_t \approx P_0(x_0)w_0 \exp[-2 \text{Im}[k_z (x_0)]z] \). The propagation distance with respect to power is given by \( L_p = (1/2) \text{Im}[k_z (x_0)] \). With the present parameters \( \phi = -0.1 \pi \) and \( d(x_0) = 10 \) \( \text{nm} \), the distance is typically \( \sim 3 \) \( \mu \)m, nearly four times the propagation distance of SPPs in a single-layer graphene. With respect to decelerated beams, the propagation loss will increase as \( x_0 \) approaches zero since the local period increases at the incident point, according to the imaginary part of \( k_z \) plotted in Fig. 1(b).

The a-GSA can also be constituted by varying the surface conductivity via chemical potential of individual graphene sheets, apart from changing the interlayer spaces (local periods). As the interlayer space is fixed at \( d \), the plasmonic thickness of graphene follows \( \xi(x) = (ax_b + 1)d \), and thus the surface conductivity abides by \( \sigma(x) = i\xi(x)\epsilon_0k_0/\eta_0 \). The variation of the surface conductivity can be realized by changing the chemical potentials of individual graphene sheets [20]. The intensity of the electric field in the a-GSA is shown in Figs. 3(a) and 3(b) for \( (a, b) = (4, -1) \) and \( (1, 2) \), respectively. The interlayer space is fixed at \( d = 20 \) \( \text{nm} \). The variation of chemical potential is shown in the inset of the figures. Similar phenomena of beam acceleration and deceleration are observed, like the cases of changing interlayer spaces.

In Fig. 4, we plot the output SPP beam positions by varying the graphene distribution density \( \alpha \) and the aperiodic degree \( b \) of the array. The incident wave is injected at \( x_0 = 1 \) \( \mu \)m for all cases. For accelerated beams \( (b = -2, -1) \), the center position of the beam is recorded at \( x = 3 \) \( \mu \)m. The increasing \( z \) position for accelerated beams indicates the decrease of acceleration. The acceleration decreases as \( a \) increases, as shown in Figs. 4(a) and 4(b). Note that the diffraction decreases at the same time according to the above discussion. For the same value of \( a \), the acceleration for \( b = -2 \) is bigger than that for \( b = -1 \). With regard to decelerated beams \( (b = 1, 2) \), the \( x \) position is recorded at \( z = 3 \) \( \mu \)m. The larger \( x \) position denotes the abatement of deceleration. The increase of \( a \) could increase the deceleration of the SPP beam, as shown in Figs. 4(c) and 4(d). At the same time, the beam width is also decreased. Enlarging value \( b \) will increase the beam deceleration as well. If the a-GSA is built by varying the chemical potential, similar results could also be achieved. The data agree well with each

Fig. 3. SPP beam routing in the a-GSA constituted by varying the chemical potential of individual graphene sheets. The interlayer space of graphene is fixed at \( d = 20 \) \( \text{nm} \). (a) Intensity of electric field of the accelerated beam as \( a = 1 \) and \( b = -1 \). (b) Decelerated beam as \( a = 1 \) and \( b = 2 \). The insets show the lateral variation of chemical potential.
other by using FDFD simulations and Hamilton optics analysis. The results clearly show the routing effect by using the a-GSAs.

In conclusion, we have studied the beam routing effects in a-GSAs constructed by varying the interlayer spaces between graphene or changing individual graphene chemical potentials. The SPPs can be accelerated and decelerated in the structures, and the propagation loss could be four times smaller than that in a single-layer graphene. The SPP beams experience almost diffraction-free propagation and the beam width could reach as small as $\lambda/130$. The wave fronts of the beams are parallel to the propagation direction, allowing for generation of transverse radiation pressure and optically manipulating small particles. The study provides a new platform for routing SPP propagation deeply below the diffraction limit.

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References

14. G. W. Hanson, J. Appl. Phys. 103, 064302 (2008).
15. A. Vakil and N. Engheta, Science 332, 1291 (2011).